THE GROWTH OF MEMORY IN SCHOOL CHILDREN.

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(From the Psychological Laboratory of Clark University.)

During the Spring of 1891, by permission of the School Board in Worcester, Mass., Dr. Franz Boas of Clark University took certain anthropological measurements of the pupils in the Grammar Schools, and also made certain tests of eye-sight, hearing and memory. The memory tests, which were made upon about fifteen hundred pupils in the Grammar Schools, above the second grade and below the high school, together with some tests from the Normal School, came into my hands for examination. To complete the material for all the grades in the public schools, the tests were made upon some of the senior and sophomore pupils in the High School.[1]

The Method of Making the Tests.—A series of numbers in which the digits were so arranged that they did not stand in their accustomed order and no digit was repeated, was read before each class to be tested, and each class was tested on four different occasions. In two Grammar Schools and in the Normal School, where the purpose was to determine the effect of fatigue, two of the tests were taken in the morning immediately after school assembled and the other two just before closing in the afternoon. In the other schools the tests were made in the morning. The digits were dictated slowly and distinctly at intervals of about two-fifths of a second with care to avoid rhythm or grouping, and at a given signal after the dictation of each number was finished, the pupils wrote the digits as they remembered them. To avoid a confusion of terms, *observation* will be used to indicate a group of five or more digits; *digits*, to designate the figures; and *place*, to indicate the position or order from

[1] I wish to acknowledge my indebtedness to Dr. Boas for this material and his advice in regard to the method of treating it. At his suggestion I have introduced the theoretical treatment of the curves. I am also indebted to Dr. E. C. Sanford for helpful suggestions, and to the teachers of the public schools who have assisted in collecting this material.

the beginning of the number—observation. In the lower grades of the Grammar Schools and in the High School twelve observations constituted a test, but in the eighth and ninth grades only nine observations were made at each test. In the lower grades the first three observations of each test were made with five digits, the second three with six; the third three with seven; and the last three with eight. In the higher grades and in the High School the first three observations were made with six digits, and in the Normal School with seven. The tests being repeated at four different times, twelve observations with five, six, seven and eight digits respectively, were made upon each pupil, and it would be possible for each pupil to get any number of the twelve correct. An observation was considered correct when only those figures which the teachers had dictated were present in the same order as that in which they had been dictated. The various kinds of errors will be treated further on. When the observations at each test are begun with five-place numbers and gradually increased to eight, the pupils easily grasp the five-place numbers and are led by these to grasp and retain more than they would otherwise be able to do. If the observations are begun with seven-place numbers and only one observation made with the seven-place number before making one with an eight or a nine, the number of figures is not so easily remembered, and more errors result from this cause. When we come to discuss the various classes of errors in the observations on the Normal School pupils, this matter will come up again. In the sixth grade of one Grammar School through a misunderstanding on the part of the teacher, all the observations taken at the first test were made with five-place numbers, those at the second with six, at the third with seven, and at the fourth with eight. After two or three trials the pupils became aware of the number of digits to expect at each test and gave their attention more to retaining the proper digits in their places. As the number of pupils is so small and the results from this grade do not differ more from the results of the next higher or lower grade than the results of some other grades differ from those of the next higher or lower grade, these pupils have been included in the charts showing the curves for ages and grades.

By classifying the pupils of a particular age or in each grade according to the proportion of the twelve observations on five-place numbers that were correct, and the same for six, seven and eight-place numbers, and marking the percentage that each class was of the total number of pupils in the grade or of the age on thirteen ordinates (twelve for the

twelve observations and one for those pupils who had none correct), and connecting these points by a line, a curve representing the distribution of the pupils of the age or grade will be obtained. The maximum of the curve will then show the proficiency of the pupils for the age or grade in remembering five, six, seven or eight-place numbers, as the case may be.

Upon Chart I the curves show the distribution of the pupils according to the ages and upon Chart II the distribution according to grades. As the observations were made with five, six, seven and eight digits, four sets of curves will be found upon each chart. The curves for the grades in which eight-place numbers were used, are found at the top of the chart, and those for seven, six and five-place numbers in order below. Under each curve appear the number of pupils and their ages on Chart I and their grades on Chart II. To give a graphic representation of the value of these curves, which is to show the increasing accuracy with which the older pupils remember a given number of digits, the average percentages of pupils in each grade and of each age, who have got six or more (for five-place numbers, ten or more) of the twelve observations correct have been taken and this average has been marked upon the twelfth ordinate of each curve. When these points are connected in each set of curves a line is obtained, the rise in which, from left to right, will then represent the increasing accuracy with which the older pupils and the pupils in the higher grades remember a given number of digits. This line, of course, is arbitrary, but it will be found to correspond very closely with the probabilities of errors for the curves. A comparison of these lines will show a more uniform rise in Chart I Chart II shows that the eighth and ninth grades fall below the seventh on eight-place numbers ; on seventh-place numbers, the fifth grade falls below the fourth, the eighth below the seventh, and the High School below the ninth grade. Other cases need not be mentioned. On Chart I only two actual falls are noticed, and these are less than one per cent. Considering that our tests measures the length of the memory-span, we can conclude that the memory-span increases with the age rather than with the growth of intelligence, as determined by the tests used in promoting pupils from one grade to another. Our tests do not apply to the retentiveness of the memory. They may be considered as tests of the power of concentrated and sustained attention. My own experience and observations upon the pupils while the tests were being made, seem to indicate that most pupils depend upon their powers of visualization to remember the number of digits, and at the

same time they were noticed to repeat the digits as they were dictated. A comparison of the standing of pupils in their grades and their ability to remember figures was undertaken to determine what was the relation between the memory-span and intellectual acuteness of the pupils. As the pupils depend upon their powers of visualization, this subject becomes more interesting in determining how far this power is of service in school work and how closely the power of concentrated and sustained attention is related to intellectual acuteness. For this purpose the teachers in the Oxford and Freeland Street Schools and of the High School were requested to give an estimation of what they considered was the general standing of their pupils with respect to the school work. The pupils were classed as either good, fair or poor, and these classes were compared with three classes which were determined in the memory test by the proportion of correct judgments. The percentages of pupils, for whom the two methods of ranking gave the same results, are given in the table. The letters of the following table (A representing good, B fair and C poor) in the left hand column indicate the teachers' classification, and those in the upper line the classification by the memory test.

TABLE I.—Giving the comparison of the teachers' estimation of standing of the pupils, and their standing, as determined by the memory tests. The classes of the teachers are represented as 100 and the others are expressed in percentages.

	A	B	C
A	32.6 %	51. %	16.3 %
B	21.4 %	58.2 %	20.4 %
C	24.1 %	49.4 %	26.5 %

Only eight and a half per cent. more of the pupils who were classed A by the teacher have been classed A rather than C by the memory tests. If our tests may be considered as tests of the ability for concentrated and sustained attention and of the power of visualization, we can conclude that these powers are not the only ones concerned in intellectual pursuits and are not sufficient for a successful undertaking of intellectual work. Intellectual acuteness, while more often connected with good powers of visualization and of concentrated attention, does not necessarily require them, and it cannot be said

that those pupils who are bright intellectually are more distinguished on account of their good memories. The fact that a good memory is not necessarily accompanied by intellectual acuteness, adds weight to the conclusion that the growth of the memory does not necessarily accompany intellectual advancement.

Theoretical Treatment of the Curves.—If we consider that the twelve observations made upon each pupil are subject to the law of chance, then we can construct from the probability of error for any curve the corresponding theoretical curve, and if our supposition is correct, the theoretical and actual curves should correspond very closely. In saying that the answers of the pupils are subject to chance we mean that they are just as likely to err on one observation as upon another, and if this is true, we can treat these observations according to the law of probability. The probability of error for any curve is obtained by subtracting the actual number of correct judgments from the possible number and finding what percentage this difference is of the possible number of correct judgments. With this percentage as the probability of error, we construct the theoretical curves according to the formula

$$p^{12}+\tfrac{12}{1}(1-p)p^{11}+\tfrac{12}{1}.\tfrac{11}{2}(1-p)^2p^{11}+\tfrac{12}{1}.\tfrac{11}{2}.\tfrac{10}{3}(1-p)^3p^{10}+\ldots.+(1-p)^{12}$$

in which p represents the probability of error. When we construct these theoretical curves, which are found in Table II, we find they do not agree except in a few cases with the actual curves. The form, however, is somewhat the same, but the absolute values are different. Either the supposition is not correct and hence this treatment is not possible, or some element has entered in this case which prevents this material from being so treated. The latter alternative is, perhaps, the correct one.

With completely naïve subjects and like conditions it does not seem probable that twelve observations upon one pupil would differ from one observation upon each of twelve pupils; and yet the variations in the probabilities for the different pupils, when classed according to age or grade, is so great that we might not get an exact correspondence between the theoretical and actual curves even with the many observations under the most uniform conditions. As we shall show further on that the children increased in their power to remember figures with each succeeding test and in one school they were not completely naïve with respect to the tests, a possible explanation is found for this disparity between the theoretical and actual curves. This explanation is further strengthened by the fact that in those curves,

where the probablity of error is less than five per cent., there is a close correspondence between the theoretical and actual curves (see Table II.). Where the probability of error is less than five per cent. for the first test, little increase in the accuracy of the judgments took place for the succeeding tests and hence the law will apply.

The actual curves are compounded of the curves represented by the probabilities of each pupil of the age or grade; the percentages of increase with each test show that there was a different probability for each observation. The effect of compounding a curve of several curves with very different probabilities is to broaden and flatten it, and it is just in this respect that the actual curves differ from the theoretical. If we construct the theoretical curves from the probabilities for the four tests on seven-place numbers given in Table VII. and compound these, we get a curve very much flatter than any of the theoretical curves. The absolute values of this curve are 0.2, 1.4, 4.7, 10., 15.3, 18.1, 17.8, 14.5, 9.8, 5.4, 2., 0.5 and 0. The probability for each test represents a curve compounded of three other curves which would have the tendancy to modify further in the same way the curves of which we have given the absolute values. This probably explains the disparity in form between the theoretical and actual curves and in view of the number of pupils represented by each curve the individual probabilities may account for the irregularities.

In Table II. the percentages for the theoretical and actual curves for the three Grammar Schools are given. The first part of the table is taken up with the actual curves, and the second with the theoretical curves. At the top of the table the numbers of the ordinates and the probabilities of error corresponding to each are given. In the columns, below the percentages of pupils for the different ages and grades and for five, six, seven and eight-place numbers are given under each ordinate for the actual curves. In the last column the figures represent the probabilities of error for the curves. For the theoretical curves these values are assumed. To make a comparison of the theoretical and actual curves take the probable error for any actual curve and find the theoretical curve whose probability of error most nearly corresponds to it. If the two curves correspond, the absolute values should agree closely.

TABLE II.— Giving the percentages of pupils and probabilities of error for the theoretical curves and for the actual curves of both the grades and the ages.

PART 1. *Actual Curves for Ages on 5, 6, 7 and 8 Digits.*

Number of Ordinate.		0	1	2	3	4	5	6	7	8	9	10	11	12	Probabilities of Error.
Probabilities of Error for each Ordinate.		% 100	% 91.6	% 83.3	% 75	% 66.6	% 58.3	% 50	% 41.6	% 33.3	% 25	% 16.6	% 8.3	% 0	
No. Digits.	Age.														
5	8 yrs.	0	0	1.	2.5	2.5	1.	4.	6.5	8.	12.	12.	14.5	34.5	20.
	9 "	0	0	2.	2.	0	1.	4.5	2.	4.5	7.5	10.	20.	45.5	14.5
	10 "	0	0	0	1.	1.	2.	1.	2.	6.5	8.5	13.	20.5	45.	12.
	11 "	0	0	0	0	0	0	1.	1.	5.	5.	10.	22.5	53.5	9.3
	12 "	0	0	0	0	0	0	3.	3.	3.	5.	8.5	18.	60.0	7.4
	13 "	0	0	0	0	0	1.	0	1.	1.	1.	17.5	18.5	58.5	6.6
	14 "	0	0	0	0	0	0	0	0	0	4.5	12.	17.	67.5	4.6
6	8 "	6.5	8.	13.	9.	9.	9.	10.5	6.5	6.5	2.5	5.	5.	8.	56.3
	9 "	7.5	6.5	9.5	6 5	9.5	5.5	3.	6.5	7.5	8.5	7.5	14.	9.5	45.5
	10 "	5.5	3.	6.5	5.5	9.5	9.5	6.5	3.	11.	9.5	9.5	15.	5.5	42.5
	11 "	1.	1.	4.	4.	5.	6.	12.	5.	13.	18.	9.5	8.5	15.	32.4
	12 "	1.	2.5	4.	2.5	4.	7.5	8.5	7.	11.5	14.5	8.5	15.	15.	31.3
	13 "	1.5	0	4.5	1.	5.5	2.5	7.	9.5	4.5	9.5	9.5	19.5	25.	27.7
	14 "	0	0	1.	1.	7.	3.	3.	12.5	10.5	11.5	14.5	11.5	25.5	23.5
	15 "	1.	1.	1.	2.5	2.5	3.5	4.5	12.	9.	11.	8.5	19.5	23.	25.3
7	8 "	40.5	16.5	14.5	2.5	0	2.	2.	2.	0	5.	0	5.	5.	78.
	9 "	15.5	8.5	8.5	14.	7.	9.5	7.	7.	8.5	4.	4.	4.	2.5	64.6
	10 "	19.5	13.	10.	8.5	9.	5.5	9.	8.5	3.5	5.5	2.	5.5	1.	66.2
	11 "	11.5	9.5	8.5	8.5	5.	9.5	11.5	7.5	6.5	6.5	6.	4.	4.	62.5
	12 "	12.5	5.	6.5	9.	5.	5.5	8.	12.5	7.5	9.	10.5	5.5	4.5	51.5
	13 "	5.5	5.5	4.5	8.	5.5	13.	8.	7.	5.5	7.	16.5	11.	3.5	49.2
	14 "	2.	5.	8.5	8.5	6.5	9.5	5.	5.	8.5	13.5	11.5	8.5	8.5	36.1
	15 "	9.5	6.5	7.5	6.5	6.5	1.	8.5	6.5	10.5	10.5	9.5	12.5	10.5	40.7
8	11 "	56.	15.5	8.	10.	2.5	2.5	5.5	1.	3.5	2.5	0	1.	2.5	84.
	12 "	23.5	15.5	12.5	5.5	6.5	4.5	8.5	4.5	3.5	4.5	2.5	2.	3.5	73.7
	13 "	25.5	17.	10.5	8.5	5.5	5.5	5.5	4.5	5.5	1.	6.5	4.5	1.	70.5
	14 "	22.5	8.5	12.5	13.5	4.5	5.5	5.5	5.5	3.	5.5	3.	7.5	3.	65.5
	15 "	18.5	12.5	7.	14.5	8.	6.5	7.	11.	4.5	6.5	2.	0	2.5	66 2

Actual Curves for Grades on 5, 6, 7 and 8 Digits.

No. Digits.	Grades.														
5	3	0	1.	1.5	3.5	3.5	1.	3.5	5.5	10.	12.5	17.5	19.5	23.	21.3
	4	0	0	0	0	0	0	1.	2.5	4.5	6.	11.	19.	57.	6.1
	5	0	0	0	0	0	1.	1.5	1.5	4.5	5.5	16.	24.	46.	8.4
	6	0	0	0	0	0	1.	0	1.5	3.	4.5	11.5	12.5	66.	6.5
	7	0	0	0	0	0	0	1.	0	0	1.	6.	15.	77.	1.6
6	3	11.	9.5	13.5	10.5	12.5	11.5	7.5	4.5	6.	2.5	3.5	3.5	3.5	63.
	4	3.5	4.	4.5	4.5	5.	7.5	5.5	5.5	7.	13.	14.	15.5	10.5	31.9
	5	1.5	4.5	7.5	5.5	9.5	8.	9.5	8.5	12.	11.	8.	11.	5.	43.
	6	1.5	1.	2.5	1.	6.5	7.5	10.5	10.5	10.5	17.5	8.5	11.5	12.5	32.
	7	0	1.	2.	3.5	2.	2.	3.5	4.5	4.5	9.	13.	23.	32.5	16.7
	8	.5	.5	2.5	0	3.	3.	6.	13.	8.5	14.5	13.	15.	20.5	16.6
	9	0	0	1.5	2.5	4.5	4.5	3.	7.	10.5	5.	8.5	22.5	32.	16.5
	H.S.	0	0	0	0	2.	2.	2.	0	6.	6.	8.	30.	46.	9.

Actual Curves for Grades on 5, 6, 7 and 8 Digits.—Continued.

	Number of Order.	0	1	2	3	4	5	6	7	8	9	10	11	12	Probable Errors.
	Probable Error of each Order.	% 100	% 91.6	% 83.3	% 75	% 66.6	% 58.3	% 50	% 41.6	% 33.3	% 25	% 16.6	% 8.3	% 0	
7	3	46.	18.	15.	4.5	3.	2.	4.5	2.	0	1.5	1.5	0	0	87.7
	4	19.	10.5	11.	8.	4.5	5.5	11.	5.	4.	8.	3.	7.	2.	63.3
	5	19.	11.	6.5	12.	7.	12.	9.5	11.	1.5	1.5	3.5	2.5	1.5	67.
	6	7.5	9.5	7.5	11.5	7.5	6.5	10.5	10.5	6.5	7.5	9.	3.5	2.5	59.
	7	6.	2.	6.	6.	5.	5.5	6.	8.5	9.5	11.	15.5	13.5	5.5	46.4
	8	6.	6.	4.5	8.5	6.	8.5	8.5	8.5	11.	9.	9.	9.	7.5	44.5
	9	3.5	4.5	5.5	3.5	3.5	2.5	4.5	5.5	7.5	13.	15.5	15.5	15.5	32.5
	H.S.	2.	4.	8.	10.	10.	10.	6.	4.	10.	12.	12.	10.	2.	50.
8	6	37.5	19.	11.	7.5	6.5	5.5	.5	1.5	2.5	1.	.5	1.5	0	85.
	7	11.	9.	12.5	13.5	6.5	5.5	8.5	6.	6.	6.	7.	6.	4.5	57.5
	8	22.5	16.	11.	12.5	6.	4.	5.5	7.	5.5	2.5	4.	1.5	2.5	70.
	9	19.	10.	8.5	12.5	5.5	7.	6.	12.	2.5	9.	1.5	2.5	3.	64.8
	H.S.	18.	18.	6.	12.	6.	10.	10.	6.	4.	4.	4.	2.	2.	67.

PART 2.—*Theoretical Curves.*

	0	0	0	0	0	0	0	0	0	.3	3.2	22.7	73.8	2.5
	0	0	0	0	0	0	0	0	.3	1.7	9.9	34.1	54.	5.
	0	0	0	0	0	0	0	.7	1.4	4.6	17.	38.1	39.3	7.5
	0	0	0	0	0	0	0	1.5	2.1	8.5	23.	37.6	28.2	10.
	0	0	0	0	0	0	.4	1.7	6.8	17.2	29.2	30.1	14.2	15.
	0	0	0	0	0	.4	1.5	5.3	13.3	23.6	28.3	20.6	6.8	20.
	0	0	0	0	0	1.1	4.	10.3	19.1	25.8	23.3	12.7	3.1	25.
	0	0	0	.1	.7	2.9	7.9	15.8	23.1	24.	16.8	7.1	1.3	30.
	0	0	0	.4	1.9	5.9	12.7	20.4	23.7	19.5	10.9	3.6	.5	35.
	0	0	.2	1.2	4.2	10.1	17.7	22.7	21.3	14.2	6.3	1.7	.2	40.
	0	.1	.6	4.3	7.2	14.9	21.3	22.3	17.	9.2	3.4	.7	.07	45.
	.02	.3	1.6	5.3	12.1	19.4	22.6	19.4	12.1	5.3	1.6	.3	.02	50.
	.07	.7	3.4	9.2	17.	22.3	21.3	14.9	7.2	4.3	.6	.1	0	55.
	.2	1.7	6.3	14.2	21.3	22.7	17.7	10.1	4.2	1.2	.2	0	0	60.
	.5	3.6	10.9	19.5	23.7	20.4	12.7	5.9	1.9	.4	0	0	0	65.
	1.3	7.1	16.8	24.	23.1	15.8	7.9	2.9	.7	.1	0	0	0	70.
	3.1	12.7	23.3	25.8	19.1	10.3	4.	1.1	0	0	0	0	0	75.
	6.8	20.6	28.3	23.6	13.3	5.3	1.5	.4	0	0	0	0	0	80.
	14.2	30.1	29.2	17.2	6.8	1.7	.4	0	0	0	0	0	0	85.
	28.2	37.6	23.	8.5	2.1	1.5	0	0	0	0	0	0	0	90.
	39.3	38.1	17.	4.6	1.4	.7	0	0	0	0	0	0	0	92.5
	54.	34.1	9.9	1.7	.3	0	0	0	0	0	0	0	0	95.
	73.8	22.7	3.2	.3	0	0	0	0	0	0	0	0	0	97.5

In Table III are given the probabilities of error for every curve upon Charts I and II. Part I of the table is taken up with the curves for ages and Part II with the curves for grades. These probable errors show the same general results that the line drawn across the curves shows. When the pupils are classified according to their ages, the figures rep-

resenting the probable errors show a more uniform decrease in passing from the younger to the older pupils than from the lower to the higher grades. The High School pupils[1] are not included in the classification for ages. Where the probabilities of error for the higher grade is greater than for a lower, or for older than for younger pupils, the number has been set in heavy faced type.

[1]The Normal School pupils have been purposely left out of this part of the treatment. The tests were not made with sufficient uniformity to allow them to be classed with the public school pupils.

TABLE III.—Probabilities of Error upon 5, 6, 7 and 8 digit series for all ages and grades.

PART 1.—*Probabilities of Error for Ages.*

No.Digits	8 yrs.	9 yrs.	10 yrs.	11 yrs.	12 yrs.	13 yrs.	14 yrs.	15 yrs.
5	20.	14.5	12.	9.3	7.4	6.6	4.6	
6	56.3	45.5	42.5	32.4	31.3	27.7	23.5	**25.3**
7	78.	**64.6**	66.2	62.5	51.5	49.2	36.1	**40.7**
8				84.	73.7	70.5	65.5	**66.2**

PART II.—*Probable Errors for Grades.*

No.Digits	3rd.	4th.	5th.	6th.	7th.	8th.	9th.	H. S.
5	21.3	6.1	**8.4**	**6.5**	1.6			
6	63.	31.9	**43.**	**32.**	16.7	16.6	16.5	9.
7	87.7	63.3	**67.**	59.	46.4	44.5	32.5	**50.**
8				85.	57.5	**70.**	**64.8**	**67.**

In psycho-physical experiments it is customary to take seventy-five per cent. of right answers as the point at which the subject may be safely said to have some knowledge of that concerning which he judges. This standard is chosen for experiments in which a choice is made between two alternatives, where, by mere guesses, the subject will get 50 per cent. correct. In our test the subject must be supposed to have exact knowledge before he can recall correctly any number of digits. Whatever standard we choose, then, for these tests, it must be considered as the probability that a certain

number of digits should be judged correctly every time. If we choose seventy-five—though it seems to me a less figure might be chosen—our tables show that all the pupils below the 6th grade and over thirteen years of age reach the limit of their memory span at six, and all others at seven. Six may then be taken as the limit to the memory span for most Grammar and High School pupils.

Any treatment of a subject of this kind would be incomplete if no comparison were made between the boys and the girls. For this purpose the boys and girls have been classified according to their ages; in order to get classes sufficiently large to form a comparison, it was necessary to put the pupils differing by two years in age instead of one into each class. The probabilities of error have been found for each class and the comparison is made in the following table. The ages together with the probabilities of correct judgments are given for each class.

TABLE IV.—Showing separately the probability of Error for Boys and Girls.

	Boys.	Girls.
FIVE-PLACE NUMBERS.		
Pupils under 10 years	16.	13.7
Pupils over 10 and under 12	10.4	11.5
Pupils over 12	11.5	11.6
SIX-PLACE NUMBERS.		
Pupils under 11 years	47.6	47.9
Pupils over 11 and under 13	35.2	26.5
Pupils over 13	25.4	25.
SEVEN-PLACE NUMBERS.		
Pupils under 12 years	65.6	61.6
Pupils over 12 and under 14	50.7	51.
Pupils over 14	51.	44.
EIGHT-PLACE NUMBERS.		
Pupils under 14 years	82.4	64.7
Pupils over 14 years	65.4	65.8

From this table it will be seen that in a majority of classes the girls make a decidedly less error than the boys. In the classes where the boys surpass the girls, it is by a very small figure. This conclusion harmonizes with the results of other observers.[1]

Unconscious Memory and Effect of Fatigue.—The tests were taken in three different Grammar Schools: Oxford, Freeland and Woodland Street Schools. In the Oxford Street School the four tests were taken in the morning and a

[1] A Statistical Study of Memory and Association, by Prof. Joseph Jastrow, Educational Review, Dec., 1891.

different series of digit-groups were used at each test. The same digits dictated at the first test were read in the inverse order at the second. They were then completely re-arranged for the third and read in the inverse order for the fourth. Thus the digits in every observation were the same for the four tests, the order alone being changed. This same arrangement was used in the tests of the Freeland Street School, two tests being taken in the morning immediately after the school assembled, and two just before closing in the afternoon. In the Woodland Street School the same digit-groups were used for all four tests, the purpose being to determine the effect of unconscious memory.

In the following Table the probabilities of correct judgments for each test on five, six, seven and eight-place numbers for all the pupils in Oxford Street School are given.

TABLE V.—Shows the probabilities of correct judgments in the Oxford Street School for the four tests with five, six, seven and eight-place numbers; 136 pupils were tested in this school.

No. Digits.	First Test. A. M.	Second Test. A. M.	Third Test. A. M.	Fourth Test. A. M.
5	82.7	91.3	83.	91.3
6	53.1	73.	71.	73.5
7	30.4	30.1	33.5	39.2
8	17.6	16.3	19.4	25.

In this school, where different series of digit-groups were used at each test, the pupils show with two exceptions considerable though not uniform increase in their ability to remember the groups of digits. This increase may be fairly taken to be the effect of practice, as the pupils remained naïve as far as possible with respect of the tests that were to be used.

TABLE VI.—Shows the probabilities of correct judgments for the Freeland Street School on four, five, six, seven and eight-place numbers. The digit groups that were used in the Oxford Street School were used in this school. Two tests were made in the morning and two in the afternoon; 219 pupils were tested in this school.

No. Digits.	First Test. P. M.	Second Test. A. M.	Third Test. P. M.	Fourth Test. A. M.
4	92.	95.	98.	95.
5	79.3	86.7	95.9	79.5
6	60.1	65.6	64.7	60.2
7	37.9	43.2	43.3	44.6
8	25.6	25.7	32.7	32.2

This Table shows that the pupils improved considerably though not uniformly with each test. They do not show greater increases for the morning than for the afternoon tests as we should expect from the fatigue of the day's work.

TABLE VII.—Showing the probabilities of correct judgments for the Woodland Street School on four, five, six, seven and eight-place numbers. The same series of digit-groups were used in all four tests. Two tests were made in the morning and two in the afternoon; 468 pupils were tested in this school.

No. Digits.	First Test. A. M.	Second Test. P. M.	Third Test. A. M.	Fourth Test. P. M.
4	96.2	97.3	98.	97.7
5	88.6	92.	94.2	94.3
6	56.7	64.4	70.1	75.5
7	40.4	50.7	58.7	64.1
8	28.4	34.9	45.9	49.7

In this school the pupils have shown uniform improvement in each test and at the same time the percentages of increase are usually larger. The morning tests do not show greater proportional increases than the afternoon.

The results from all the schools point to the conclusion that the pupils improve with practice. The great uniformity and large increases with each test in the Woodland Street School seem to show that the pupils unconsciously remember the digits that have been dictated one day previous. The probabilities of correct judgment do not show any variations due to fatigue. The total number of correct judgments for the morning tests in the Freeland Street School are 2,69 and for afternoon tests 2,640; for the morning tests in Woodland Street School 6,609, and for the afternoon tests 7,179. When we consider that great increases were made with each test, and the first test in the Freeland Street School was made in the afternoon, we should expect a greater number of correct judgments for the morning test; and since the first test was made in the Woodland Street School in the morning, we should expect a greater number of correct judgments for the afternoon test. This is just what the figures show, and we may safely conclude that the pupils suffer no fatigue from their school work, at least none discoverable by such tests as these. Their work is probably not excessive.

The Nature of Errors.—A careful examination of the observations shows that there were three, perhaps four, classes of errors which represent stages in the fading of the memory-image. In the first stage the digits suffer a displacement of order; in the second, other digits are substituted for some that were dictated and in the third, the number of digits is misjudged, either over- or under-estimated. Various causes may be assigned for the displacement of order. When the pupil attempts to write, the attention passes over the successive digits in memory as a rule much faster than they can be written. Before the pupils can write the first digit, the attention has passed to the third or fourth and the hand is innervated for the digit that is present in consciousness. The second may be immediately recalled and is put in the third place. It more frequently happens that the fourth or fifth is displaced than the second or third. Again, the order of the digits in the numbers previously dictated clings in the mind and causes the figures in the next number to be interchanged in accordance with that order.[1] A single case will be sufficient to make this statement clear. Two numbers, the first commencing with 8163 and the second with 5136, were dictated. The 3 and 6 in the second were frequently reversed so as to read 63. Further, the order in which the digits stand in our system of notation

[1] Dr. Leo Bergerstein, Zeitsch. f. Schulgesundheitspflege, No. IX and X. 1891.

determines some changes. One case in particular deserves mention and will be of service to others who attempt any work in this line. The last three digits of one number were 768 and a very frequent error was to change the order of the 7 and 6 to 67. It is probable also that when these digits were read in the inverse order, 867, the order of the 6 and 7 is again changed to read 876. In other cases a digit seems to have become associated with one place in the number from having frequently occupied that place, and when this digit appears in the next succeeding number, it changes places with the one that occupies the position it has become associated with; even when it does not appear in the following number, it may be substituted for one that occupies its associated position. In many cases it is very difficult to determine what has brought about the change and whether the error is an error of inversion or substitution. Number habits and the association of one digit with another from some experience in life—the number of the house, the year or day of the month of a child's birth—would seem to enter as factors. A fact that has been noticed frequently in teaching children and also adults is the great liability to confusion, when it is attempted to keep separate two like organs whose functions are diametrically opposed. In physiology it is a difficult matter for children to distinguish the functions of the right and left ventricles of the heart and even for adults the functions of the dorsal and ventral columns of the spinal cord. It seems probable that this difficulty also appears in keeping the order of two digits that are easily remembered. 1 and 5 standing at the end of a number, where the digits are rarely forgotten were frequently interchanged. The inversion of the order is by far the most frequent error, as it is also the first to occur.

In substituting a new digit for one that has been read, there enter some of the causes that bring about an inversion of the order. A digit is substituted for another to make the two stand in the order they do in our system of notation, or in the order in which they were in the number previously dictated. The likeness in the sounds of the names of two digits often determines the substitution in the one for the other. Nine and five and nine and one are frequently interchanged. The written or printed forms of 9 and 1 probably have something to do with the substitution of the one for the other. The very frequent interchange of 3 for 8 is due unquestionably to the likeness in the form of the printed digits. The likeness in the innervation required for two digits would seem to explain the substitution of 5 for 3 and 7 for 9. Substitution stands next in frequency to inversion of the order.

When the digits are left out the pupils more frequently have forgotten the proper digits and also their associations and so drop them out altogether. Whatever may be the cause of the dropping of a digit, the fact that it is left out shows a more advanced stage in the disappearance of the memory-image. The places in which the most errors of every kind are likely to occur are the positions from which digits are most frequently dropped. In some cases it seems probable that a digit may be dropped from the tendency to bring two associated digits in juxtaposition or two digits that stand juxtaposed either in our system of notation or in some number previously dictated.

When the pupils overestimate the number of digits, two tendencies only were noticed. The digits that were supplied were put in the places in our system of notation that occur between some two digits already given, or they were placed between two digits which should stand together and which were separated by the supplied digits in some number previously dictated. When two digits already stand in their natural order, the tendency is very strong to put another digit in order either before or after those given. The second tendency was to repeat some digit already written. Over-estimations of the number are very infrequent, probably for the reason that each test was begun with numbers that could be easily grasped and digits that could be counted. In the Normal School the observations were begun with seven-place numbers; but instead of making three observations, as was done in the Grammar Schools, with seven digits, the teacher dictated only one seven-place number before dictating an eight- and a nine-place number. Again, a seven- and an eight- and a nine-place number were dictated and so on until fifteen observations were made at each test. As the curves have shown that six figures are all that the best pupils can easily span, the Normal School pupils were taxed to the limit of their powers on the first trial. In the Grammar Schools the pupils were started with numbers they could easily grasp and were led by steps to expect the number of digits in each succeeding observation. On this account 180 pupils from the Oxford Street School over-estimated the number of digits on observations with seven- and eight-place numbers 88 times; and 24 Normal School pupils over-estimated the number of digits 76 times. In counting the errors that arise from dropping digits no separate account was taken of the cases where the pupils dropped the digits because they did not remember the number of digits given, and where they dropped them because they failed to recall the correct digits. In most cases it would be difficult to determine this. This

tendency, however, to over-estimate shown by the Normal School pupils is not the general rule, as the experiments of other observers have pretty conclusively shown. Drs. Hall and Jastrow[1] found that the tendency was to under-estimate the number of clicks made by a quill held against the notched circumference of a revolving wheel, when the number of clicks was too rapid to be counted. Other experiments point in the same direction. It would seem then that at the moment when pupils reached the limit of their memory span, they over-estimated the number of digits; but if the experiments had been continued with a greater number still, the pupils would have under-estimated the number.

Method of Correcting the Observations and Counting the Errors.—The method of correcting and counting the errors has an important bearing upon the number and the classes and so must be given in some detail. Any treatment of the errors will necessarily be more or less arbitrary; but that treatment which gives the most uniform results and can be the most consistently applied would seem to be the least arbitrary. Some account too must be taken of the number of errors. The least possible number of changes necessary to make a given number correct was made, and each change was counted an error and classed an error according to its kind. The numbers were corrected as far as possible by restoring the order, before resorting to the substitution of other digits. The digits not required were then cut out and the proper substitutions made, the last process being to supply the vacant places with digits. It would be possible, however, to correct any number by either cutting out digits or substituting others. This would reduce the number of classes of errors but would greatly increase their number and destroy the distinction between the classes, which is based upon three different psychological processes. After careful consideration this method seemed to give the most uniform results and was the easiest to apply consistently.

The Frequency of the Different Classes of Errors.—To determine the class of errors which was first to occur, all those observations in which only one kind of error was made, were examined on 105 tests from the fourth and fifth grades.

[1] Studies of Rhythm, by Professor G. Stanley Hall and Joseph Jastrow. Mind, Volume XI., No. 41.

TABLE VIII.—Showing the different classes of errors.

	Inversions of Order.	Substitutions.	Over Estimations.	Under Estimations.
Five-place Numbers . .	10	7	3	1
Six-place Numbers . .	44	15	15	5
Seven-place Numbers .	40	26	13	2
Eight-place Numbers .	33	10	14	4

If we take observations with a much greater number of digits, we should expect that the greatest number of errors would result from the dropping of digits. This matter will be discussed again further on. The total numbers of errors of every class in the Oxford Street School, when compared with the total number of observations, give the following percentages: Inversions of the order, 53%; substitutions, 31%; under and over estimations of the number of digits 15%.[1]

Position of the Error.—In what places are errors most likely to occur, or in what places are the digits most often forgotten? We encounter here a new difficulty in deciding which digits are incorrect. This difficulty applies only to digits that are interchanged. Then, too, when the pupils have left out figures they frequently indicated them by leaving a blank space. The first three digits are frequently present in their proper order, then follows a blank space and the last two in their order. The last three may be present and the rest absent or *vice versa*. To avoid this difficulty, all the proper digits that stood in their proper order either from the beginning or the end of the number, were considered correct and all others incorrect. 300 observations from the

[1]Important in this connection are the experiments of Dr. H. Münsterberg, *(Die Association successiver Vorstellungen*, Zeitsch. f. Psychol. Bd. I., H. 2, 1890.) He found that he could repeat seven letters without error, when they had been exposed one at a time in such a way that each was seen for one second; but he reached the limit of his power at ten. The most frequent error was that of substitution; inversions of the order were very rare. When, however, he solved problems in mental arithmetic aloud, while the letters were being exposed, his upper limit fell to seven, and only four or five could generally be recalled correctly. Instead of errors of substitution simply, errors of inversion of the order became the more frequent. The explanation he offers for this does not seem in the light of these tests to be conclusive. Possibly the difference in rate at which numbers and letters were given may help to account for the difference.

senior class in the High School on six, seven, eight and nine-place numbers and 300 from the eighth and ninth grades of Freeland and Woodland Street Schools have been examined with reference to the place in which the errors were most likely to occur. The errors occurring in each place were counted and compared with the possible numbers. The percentages were marked upon six ordinates for six-place numbers, on seven for seven-place numbers and so on, and these points connected by lines. The curves thus obtained will represent the relative frequency with which an error occurs in each place. Chart III. shows the curves for the Grammar School grades and for the High School. Those for the Grammar School are represented by the unbroken line and those for the High School by the dotted line. All the curves show a gradual though not uniform rise from the first place to one place past the middle; the curves then fall at first slowly, then rapidly, until they reach a point in the last place almost as low as at the beginning, in two cases lower. If observations with more than nine digits were made, we should expect that the pupils would be likely to forget the first more often than the last, so that it is not a mere accident that the curve for nine-place numbers ends lower than it begins. If fifteen or more digits were made the subject of observation, the pupils would probably forget all but the last two or three with only an occasional recollection of the fourth from the last and the first. There is a backward flow to the memory from the last, which affects quite perceptibly the second, slightly the third and in individual cases the fourth from the end. We have here a demonstration of the well-known rhetorical principle that the emphatic words in a sentence are the first and the last. In addition to this we get an idea of the relative importance of the other words. This will be true of any series of successive ideas. They are permanent in an inverse order as they are removed from the beginning except the last two or three which are permanent in their order from the last.

Conclusions.—I. The limit to the memory span for the pupils in the public school is six.

II. The memory-span increases with age rather than with the growth of intelligence. Experience in this matter is a better school than books.

III. The memory-span measures the power of concentrated and prolonged attention.

IV. Intellectual acuteness, while more often accompanied by a good memory-span and great power of concentrated and prolonged attention, is not necessarily accompanied by them.

V. The girls have better memories than the boys.

VI. With practice, pupils increase in their ability to remember groups of digits.

VII. Pupils unconsciously remember digits that they heard a day before, when they are used second time.

VIII. The tests do not show that the pupils suffer fatigue from the day's work. This fact shows that the work in the schools is probably not excessive.

IX. Memory images pass through three stages in leaving the mind. First, they suffer a confusion of order; second, a loss of certain elements and the substitution of associated elements; and third, a complete loss of some elements and no recovery.

X. Ideas previously in the mind and association forms of ideas are factors in causing the confusion of the memory image and its final loss.

XI. There is an apparent tendency to over-estimate the number of ideas presented to the mind, when the number of ideas is slightly greater than the memory span; but the general rule is to under-estimate the number.

XII. Ideas, except the last two or three in a series, are lasting in an inverse order as they are removed from the beginning of the series in which they occur. The last two or three are lasting but in decreasing degree as they are removed from the end of the series.

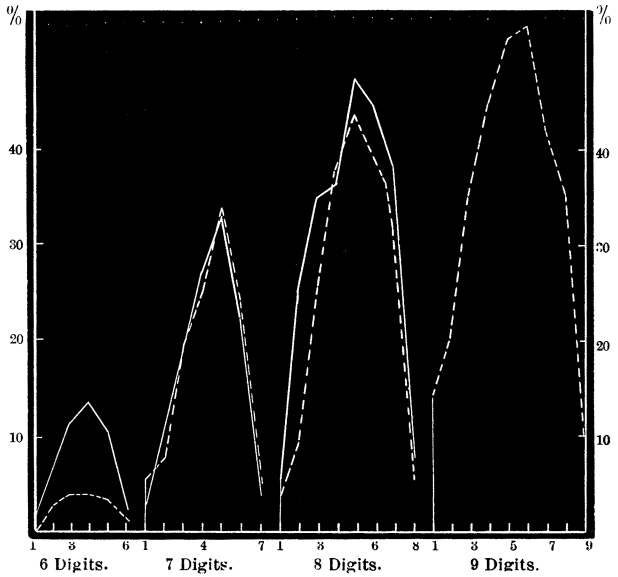

CHART III.—Shows the relative liability of a digit to be forgotten in any order in six, seven, eight and nine-place numbers. The unbroken line represents the distribution of errors for eighth and ninth grade pupils, and the broken line the distribution for High School pupils.